

Implied Recovery Rates—Auctions and Models

Stephan Höcht, Matthias Kunze and Matthias Scherer

Abstract Credit spreads provide information about implied default probabilities and recovery rates. Trying to extract both parameters simultaneously from market data is challenging due to identifiability issues. We review existing default models with stochastic recovery rates and try calibrating them to observed credit spreads. We discuss the mechanisms of credit auctions and compare implied recoveries with realized auction results in the example of Allied Irish Banks (AIB).

1 Introduction

Corporate credit spreads contain the market's perception about (at least) two sources of risk: the time of default and the subsequent loss given default, respectively, the recovery rate. Default probabilities and recovery rates are unknown parameters—comparable to the volatility in the Black–Scholes model. We concern the question whether it is possible to reverse-engineer and disentangle observed credit spreads into these ingredients. Such a reverse-engineering approach translates market values into model parameters, comparable to the extraction of market implied volatilities in the Black–Scholes framework. There is growing literature in the field of implied default probabilities, whereas scientific studies on implied recoveries are sparse. Inferring implied default probabilities from market quotes of credit instruments often relies on the assumption of a fixed recovery rate of, say, $\Phi = 40\%$. Subsequently,

S. Höcht (✉)

Assenagon GmbH, Prannerstrasse 8, 80333 Munich, Germany
e-mail: stephan.hoecht@assenagon.com

M. Kunze

Assenagon Asset Management S.A., Zweigniederlassung München, Prannerstrasse 8,
80333 Munich, Germany
e-mail: matthias.kunze@assenagon.com

M. Scherer

Lehrstuhl für Finanzmathematik, Technische Universität München, Parkring 11,
85748 Garching-Hochbrück, Germany
e-mail: scherer@tum.de

© The Author(s) 2015

K. Glau et al. (eds.), *Innovations in Quantitative Risk Management*,
Springer Proceedings in Mathematics & Statistics 99,
DOI 10.1007/978-3-319-09114-3_9

default probabilities are chosen such that model implied credit spreads match quoted credit spreads. The assumption of fixing $\Phi = 40\%$ is close to the market-wide empirical mean (compare Altman et al. [1]), but disregards recovery risk. In many papers, the same recovery rate is assumed for all considered companies, although empirical studies suggest that recoveries are time varying (compare Altman et al. [2], Bruche and González-Aguado [3]), depend on the specific debt instrument, and vary across industry sectors (compare Altman et al. [1]). Obviously, the resulting implied default probability distribution strongly depends on the assumptions on the recovery rate. Since default probabilities and recoveries both enter theoretical spread formulas, we face a so-called identification problem. Making this more plastic, the widely known approximation via the “credit triangle” (see, e.g., Spiegeleer et al. [4, pp. 256]) suggests:

$$\text{spread } s = (1 - \Phi)\lambda, \quad (1)$$

where Φ is the recovery rate and λ denotes the default intensity. Obviously, for any given market spread s , the implied recovery is a function of (the assumption on) λ and vice versa. Using this simplified spread formula alone, it is clearly impossible to reverse-engineer Φ and λ simultaneously from s . As we will see, this identification problem also appears in more sophisticated credit models.

We invoke and (at least partially) answer the questions:

- Is it possible to simultaneously extract implied recovery rates and implied default probabilities (under the risk-neutral measure \mathbb{Q})?
- How do implied recoveries compare to realized recoveries?¹

We address the first question using two types of credit models, where neither the recovery rate nor the default probability distribution is fixed beforehand. As opposed to most existing approaches for the calculation of implied recoveries, both procedures only take into account prices from simultaneously traded assets. Instead of analyzing the spread of one credit instrument for different points in time, implied recoveries and default probabilities are extracted from the term structure of credit spreads. Likewise to the aforementioned implied volatility calculation, this restriction allows for an implied recovery calibration under the risk-neutral measure \mathbb{Q} . Analyzing the second question, both models are exemplarily calibrated to market data of Allied Irish Banks (AIB), who experienced a credit event in June 2011. Subsequently, real recovery rates were revealed and can thus be compared to their implied counterparts. In order to clarify how real recoveries are settled in today’s credit markets, we start by introducing the mechanism of credit auctions.

¹ Here, the term realized recovery does not refer to workout recoveries but to a credit auction result. The question whether the auction procedure appropriately anticipates workout recoveries is left for future research.

2 CDS Settlement: Credit Auction

CDS are the most common and liquidly traded single-name credit derivatives—their liquidity usually even exceeds the one of the underlying bond market. In case of a credit event, the protection buyer receives a default payment, which approximates the percentage loss of a bond holder subject to this default² (see Schönbucher [5, preface]). This payment is referred to as loss given default (LGD). The corresponding recovery is defined as one minus the LGD. Recoveries are often quoted as rates, e.g., referring to the fraction of par the protection buyer receives, after the CDS is settled. There are mainly three types of credit events that can be distinguished:

- **Bankruptcy** A bankruptcy event occurs if the company in question faces insolvency or bankruptcy proceedings, is dissolved (other than merger, etc.), liquidated, or wound up.
- **Failure-to-pay** This occurs if the company is unable to pay back outstanding obligations in an amount at least as large as a prespecified payment requirement.
- **Restructuring** A restructuring event takes place if any clause in the company's outstanding debt is negatively altered or violated, such that it is legally binding for all debt holders. Not all types of CDS provide protection against restructuring events.

These credit events are standardized by the International Swaps and Derivatives Association (ISDA). The legally binding answer to the question, whether or not a specific credit event occurred, is given by the so-called Determinations Committees (DC).³ CDS ISDA standard contracts as well as the responsible DCs differ among geopolitical regions. As opposed to standard European contracts, the standard North American contract does not provide protection against restructuring credit events. The differences are originated by regulatory requirements and the absence of a Chapter 11 equivalent: in order to provide capital relief from a balance sheet perspective, European contracts have to incorporate restructuring events. Our focus will be on the case of nonrestructuring credit events in what follows.

Prior to 2005, CDS were settled physically, i.e., the protection buyer received the contractually agreed notional in exchange for defaulted bonds with the same notional. Accordingly, the corresponding CDS recovery rate was the ratio of the bond's market value to its par. This procedure exhibited different shortfalls (see Haworth [6, p. 24] or Creditex and Markit [7]):

- For a protection buyer, it was necessary to own the defaulted asset. Often, this entailed an unnatural inflation of bond prices after default and became a substantial

² We will use “credit event” and “default” as synonyms. Note, however, that the terms default and credit event are sometimes distinguished in the sense that default is associated with the final liquidation procedure.

³ More information on DCs and ISDA can be found on www.isda.org.

problem in default events, where the notional of outstanding CDS contracts exceeded the par of available bonds by multiples.⁴

- On the contrary, the protection seller was obliged to own the defaulted asset after settlement of the CDS. Thus, she or he mandatorily retained a long position with respect to the reference entity's credit risk, making it less attractive to sell protection.
- Since different bonds generally may have different prices, there was no unique settlement price and two identical CDS contracts often were settled against different recoveries, depending on the liquidity of the associated bond market.

These shortfalls were the initial motivation to alter the standard settlement procedure by introducing an auction-based method. From 2005 to 2013 auctions for the settlement of CDS and LCDS (Loan Credit Default Swaps) contracts for 112 default events were held (see Creditex and Markit [8]). On an annual basis, the number of auctions clearly peaked after the financial crisis, i.e., in 2009, where auctions for 45 default events took place. The recovery of a standard CDS contract, traded today, thus usually refers to the result of an auction, which is held subsequent to a credit event.

The auction mechanism aims at a unique and fair settlement price (recovery). It can be split into two stages: the initial bidding period and a subsequent one-sided Dutch auction. The whole process is administrated by Creditex and Markit. In the initial bidding period, each participant, i.e., each protection seller or buyer, represented by one of the bigger investment banks as their dealer, submits a two-way quote. This quote consists of a bid and an offer price for the cheapest-to-deliver bond of the reference entity together with a one-way physical settlement request. In the one-sided Dutch auction, the unique recovery for all outstanding CDS is assessed as the "fair" value of the cheapest-to-deliver bond with respect to its par.⁵ Before the auction starts, a quotation amount, a maximum bid-offer spread, and the cap amount is published by ISDA. These three quantities will be explained, while passing through the auction.

2.1 Initial Bidding Period

All participants submit a two-way quote together with a one-way physical settlement request. That quote refers to the price of the cheapest bond which is listed as deliverable obligation by ISDA. The request must be in the same direction as the net CDS position, e.g., participants that have net sold protection are not allowed to request delivery of an obligation. Furthermore, the two-way quote must not violate the maximum bid-offer spread. In case a dealer does not represent any outstanding CDS positions with respect to the defaulted entity, she or he is not admitted to

⁴ Sometimes the phenomenon that some bonds were used several times for the settlement of CDS is referred to as "recycling."

⁵ Restructuring events differ, since they allow for maturity specific cheapest-to-deliver bonds.

participate in the auction. Moreover, the notional of the physical settlement request is not allowed to exceed the notional of the outstanding position.

In the next step, the so-called inside market midpoint (IMM) is calculated subject to the following method:

1. Crossing quotes are canceled, i.e., in case an offer quote is smaller or equal to another bid quote, the specific bid and offer are both eliminated.⁶
2. The so-called best halves of the remaining quotes are constructed. The best bid half refers to the (rounded up) upper half of the remaining bid quotes. Accordingly, the best offer half contains the same number of lowest non-canceled offer quotes.
3. The IMM is defined as the average of all quotes in those best halves.

Any participant, whose bid and ask price are both violating the IMM has to pay an adjustment amount.⁷ This penalty is supposed to assure that the IMM reflects the underlying bond market in an appropriate way.⁸ The initial bidding period is concluded by calculating the net open interest, i.e., the netted notional of physical settlement requests, which is simply carried out by aggregation. In case this amount is zero, the IMM is fixed as the auction result and consequently as the recovery for all CDS, which were supposed to be settled via the auction. Otherwise, the IMM serves as a benchmark for the second part of the auction procedure.

To illustrate this first step, we consider the failure-to-pay event of AIB on June 21, 2011. Two auctions were held, one for senior and one for subordinated CDS referring to AIB. We only consider the senior auction. Table 1 displays the submitted two-way quotes from all 14 participants. For the calculation of the IMM, the reported bid quotes are arranged in descending order, whereas the offers start from the lowest quote.

The first quotes from Nomura (bid) and Citigroup (offer) are canceled out, since the corresponding bid exceeds the offer. Note that this cancelation does not entail a settlement, both quotes are merely neglected with regard to the IMM calculation. Therefore, 13 bid and offer quotes remain and the best halves are the seven highest bid and lowest offer quotes, which are emphasized in Table 1. The IMM is calculated via averaging over these quotes and rounding to one eighth, yielding an IMM of 71.375. The maximum bid-offer spread was 2.50 %-points and the quotation amount was EUR 2 MM. In Table 2, the corresponding physical settlement requests are reported.

As the aggregated notional from bid quotes exceeds the aggregated notional from offer quotes, the auction type is “to buy”. Since there is netted demand for the cheapest-to-deliver senior bond, initial offers falling below the IMM are considered

⁶ Note that they are not settled, but only not taken into account for the calculation of the IMM.

⁷ The term “violating” refers to both quotes falling below the IMM (auction is “to buy”) or exceeding the IMM (auction is “to sell”), respectively.

⁸ Suppose the net open interest is “to sell”, i.e., there is a surplus on the seller side. If a participant submits a bid exceeding the IMM, he or she is considered off-market, since prices are supposed to go down and not up. Then the corresponding participant has to pay the prefixed quotation amount times the difference between the IMM and his or her bid. The penalty works vice versa for off-market offers if the open interest is “to buy”.

Table 1 Dealer inside market quotes for the first stage of the auction of senior AIB CDS (see Creditex and Markit [8]). Published with the kind permission of ?Creditex Group Inc. and Markit Group Limited 2013. All rights reserved

Dealer	Bid	Offer	Dealer
Nomura Int. PLC	72.00	70.50	Citigroup Global Markets Ltd.
Goldman Sachs Int.	71.00	71.50	Société Générale
Bank of America N.A.	70.50	72.00	Credit Suisse Int.
Barclays Bank PLC	70.50	72.00	Deutsche Bank AG
BNP Paribas	70.50	72.00	JPMorgan Chase Bank N.A.
HSBC Bank PLC	70.50	72.25	Morgan Stanley &Co. Int. PLC
The Royal Bank of Scotland PLC	70.50	72.50	UBS AG
Deutsche Bank AG	70.00	73.00	Bank of America N.A.
UBS AG	70.00	73.00	Barclays Bank PLC
Morgan Stanley &Co. Int. PLC	69.75	73.00	BNP Paribas
Credit Suisse Int.	69.50	73.00	HSBC Bank PLC
JPMorgan Chase Bank N.A.	69.50	73.00	The Royal Bank of Scotland PLC
Société Générale	69.00	73.50	Goldman Sachs Int.
Citigroup Global Markets Ltd.	68.00	74.50	Nomura Int. PLC
Resulting IMM	71.375		

All quotes are reported in %

Table 2 Physical settlement requests for the first stage of the auction of AIB (see Creditex and Markit [8]). Published with the kind permission of ?Creditex Group Inc. and Markit Group Limited 2013. All rights reserved

Dealer	Type	Size in EUR MM
BNP Paribas	Offer	48.00
Credit Suisse Int.	Offer	43.90
Morgan Stanley &Co. Int. PLC	Offer	11.80
Barclays Bank PLC	Bid	30.00
JPMorgan Chase Bank N.A.	Bid	52.00
Nomura Int. PLC	Bid	7.75
UBS AG	Bid	16,00
Total (net)	“To buy”	2.05

off-market and the corresponding dealers have to pay an adjustment amount. In Table 1, only Citigroup’s offer of 70.50 is considered off-market. The difference to the IMM is 0.875. Using the quotation amount as notional, the resulting adjustment amount is EUR 17, 500. The second part of the auction aims at satisfying the net physical settlement request of EUR 2.05 MM demand.

2.2 Dutch Auction

This second step is designed as a one-sided Dutch auction, i.e., only quotes in the opposite direction of the net open interest are allowed. In case the net open interest is “to sell”, dealers are only allowed to submit bid limit orders and vice versa. For the senior CDS auction of AIB, the net physical settlement request is “to buy” and thus only offer limit orders are allowed. As opposed to the first stage of the auction, there is no restriction with respect to the size of the submitted orders, regardless of the initial settlement request. In order to prevent manipulations, particularly in case of a low net open interest, the prefixed cap amount, which is usually half of the maximum bid-offer spread, imposes a further restriction on the possible limit orders. In case the auction is “to sell”, orders are bounded from above by the IMM plus the cap amount and vice versa if the net open interest is “to buy”.

In addition to these new limit orders, the appropriate side from the initial two-way quotes from the first stage of the auction are carried over to the second stage—as long as the order does not violate the IMM. All quotes, which are carried over, are determined to have the same size, i.e., the prespecified quotation amount, which was already used to assess the adjustment amount.

Now, all submitted and carried over limit orders are filled, until the net open interest is matched. In case the auction is “to sell”, i.e., there is a surplus of bond offerings, the bid limit orders are processed in descending order, starting from the highest quote. Analogously, if the auction is “to buy”, offer quotes are filled, starting from the lowest quote. The unique auction price corresponds to the last quote which was at least partially filled. Furthermore, the result may not exceed 100 %.⁹

Reconsider the credit event auction for outstanding senior AIB CDS. Both, carried over offer quotes (first) as well as offers from the second stage (second) of the auction are reported in Table 3.

Recalling that the net physical settlement request was EUR 2.05 MM, we observe that the first two orders were partially filled. The associated limit orders were 70.125 %, which is consequently fixed as the final auction result, i.e., all outstanding senior CDS for AIB were settled subject to a recovery rate of 70.125 %. Following an auction, all protection buyers, who decided to settle their contracts physically beforehand, are obliged to deliver one of the deliverable obligations in exchange for par. Naturally, they are interested in choosing the cheapest among all possible deliverables. Thus, in case of a default, protection buyers are long a cheapest-to-delivery option (compare, e.g., Schönbucher [5, p. 36]), enhancing the position of a protection buyer. Details about the value of that option can be found in Haworth [6, pp.30–32] and Jankowitsch et al. [9].

⁹ For Northern Rock Asset Management, the European DC resolved that a restructuring credit event occurred on December, 15, 2011. Two auctions took place on February, 2, 2012 and the first one theoretically would have led to an auction result of 104.25 %. Consequently, the recovery was fixed at 100 % (compare Creditex and Markit [8]).

Table 3 Limit orders for the senior auction of AIB (see Creditex and Markit [8]). Published with the kind permission of Creditex Group Inc. and Markit Group Limited 2013. All rights reserved

Dealer	Type	Quote (%)	Size (EUR MM)	Aggregated size (USD MM)
JPMorgan Chase Bank N.A.	Second	70.125	2.05	2.05
Barclays Bank PLC	Second	70.125	2.05	4.10
Credit Suisse Int.	Second	70.25	2.05	6.15
BNP Paribas	Second	70.25	1.00	7.15
BNP Paribas	Second	70.375	1.05	8.20
Citigroup Global Markets Ltd.	First	71.375	2.00	10.20
			...	
Nomura Int. PLC	Second	75	2.00	42.25

2.3 Summary of the Auction Procedure

The auction-based settlement of CDS is designed to approximate the loss of the cheapest-to-deliver bond. The term “CDS auction” might thus be misleading, since it is an auction, where the market value of the cheapest from a set of bonds is assessed. Consequently, the recovery rate of a CDS contract is the market value of this bond divided by its par.

In the above example, JPMorgan’s and Barclays’ orders were the only ones filled. Both dealers had a considerable physical settlement request of EUR 52 MM and EUR 30 MM, respectively, possibly reflecting a long CDS position. By submitting the lowest possible quote for a notional of EUR 2.05 MM each, both dealers stretched the recovery to the possible maximum. In case, both parties indeed represented large long CDS positions, they profited from the low open interest. Moreover, the final auction result was below the IMM. Thus, if one dealer would have quoted the final auction result already in the first step, she or he would have been considered off-market and consequently penalized.

Another problem appeared during a restructuring credit event of SNS bank, where senior and subordinated CDS were settled in the same auction. Due to government intervention, subordinated bond holders experienced a full write-down (“bail-in”) before the auction. Thus, there were no more subordinated deliverables and senior and subordinated CDS had the very same recovery (either 95.5 or 85.5 %, depending on the maturity of the CDS), contradicting the connection between the subordinated bond holder’s loss and the subordinated CDS recovery. Another case for a counterintuitive auction result concerned the settlement of CDS referring to Fannie Mae or Freddy Mac, where subordinated contracts recovered above senior. Moreover, as the determination committees and dealers are big investment banks, there might be conflicts of interest when determining whether a credit event occurred or not.

These are reasons for an ongoing discussion about whether this one-sided auction design is fair or not (compare Du and Zhu [10] for the proposal of an alternative

auction design). Currently, ISDA is working on a further supplement to the credit derivative definitions, involving among others the introduction of a new credit event as a solution to what happened with subordinated SNS CDS.

3 Examples of Implied Recovery Models

As explained above, the recovery of a CDS, $\Phi_\tau \in [0, 1]$, refers to the result of an auction which is held after a credit event at time τ and is designed to approximate the relative “left-over” for a bond holder. Before a default event and the following auction takes place this recovery is unknown. One way to assess this quantity for nondefaulted securities is to reverse-engineer implied recoveries from market CDS quotes. Any basic pricing approach for the “fair” spread s_T of a CDS with maturity $T > 0$ is of the form

$$s_T = \mathbb{E}_{\mathbb{Q}}[f(\tau, \Phi_\tau)]. \quad (2)$$

I.e., the spread is the risk-neutral expectation of a function of the default time (or default probability, respectively) and the recovery rate in case of default. Specifying τ and Φ_τ , two models are revisited and calibrated by minimizing the root mean squared error (RMSE) between $\mathbb{E}_{\mathbb{Q}}[f(\tau, \Phi_\tau)]$ and market spreads over a term structure of CDS spreads.

3.1 Cox–Ingersoll–Ross Type Reduced-Form Model

This reduced-form model resembles the one presented in Jaskowski and McAleer [11], although applied in a different context. All reduced-form models are based on the same principle. The time of a credit event τ is the first jump of a stochastic counting process $Z = \{Z_t\}_{t \geq 0} \in \mathbb{N}_0$, i.e., $\tau = \inf \{t \geq 0 : Z_t > 0\}$. In this case Z will be a Cox-Process governed by a Cox–Ingersoll–Ross type intensity process λ , i.e.,

$$d\lambda_t = \kappa(\theta - \lambda_t)dt + \sigma\sqrt{\lambda_t}dW_t, \quad \lambda_0 > 0.$$

The recovery in this model is defined as an exponential function of the intensity process, i.e.,

$$\Phi_\tau := ae^{-\frac{1}{\tau} \int_0^\tau \lambda_t dt},$$

where $a \in [0, 1]$ is referred to as the recovery parameter. A default in a period of high expected distress, e.g., in an economic downturn, entails lower recoveries

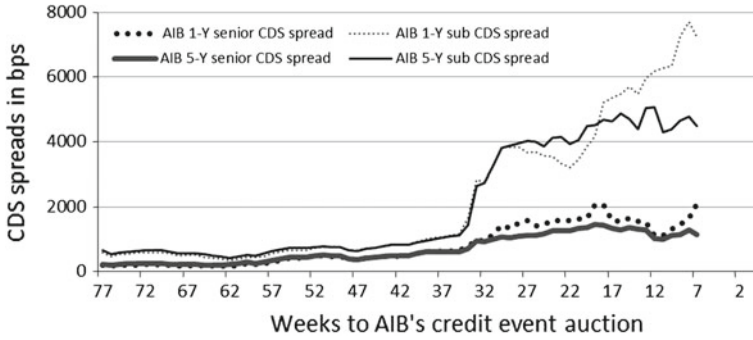


Fig. 1 Weekly average spreads for AIB senior and subordinated CDS with 1 and 5 years maturity. The spreads represent two whole term structures, which are used to calibrate the presented implied recovery approaches in every displayed week independently

and vice versa. Comparable choices for modeling recoveries can be found, e.g., in Madan et al. [12], Das and Hanouna [13], Höcht and Zagst [14], or Jaskowski and McAleer [11]. Since the model will be calibrated to one CDS spread curve, one has to be restrictive concerning the amount of free model parameters in the recovery model. Using this model, the risk-neutral spread $s_T(\kappa, \theta, \sigma, \lambda_0, a)$ has an integral-free representation. The resulting risk-neutral parameters and subsequently the risk-neutral implied recovery and probability of default are determined by minimizing the RMSE:

$$(\kappa^*, \theta^*, \sigma^*, \lambda_0^*, a^*) := \operatorname{argmin} \sqrt{\frac{1}{|\mathbb{I}|} \sum_{T \in \mathbb{I}} (s_T^M - s_T(\kappa, \theta, \sigma, \lambda_0, a))^2}, \quad (3)$$

where \mathbb{I} is the set of maturities with observable market quotes for CDS spreads s_T^M . In case senior as well as subordinated CDS are available for a certain defaultable entity, two different recovery parameters a_{sen} and a_{sub} are used, while the intensity parameters are the same for both seniorities. This reflects the fact that in case of a credit event both CDS types are settled, although usually in different auctions.¹⁰ In this case, the optimization in Eq. (3) is simply carried out by matching senior and subordinated spreads simultaneously. For the calibration, we reconsider the example of AIB. Figure 1 exemplarily shows weekly average quotes for AIB senior and subordinated CDS spreads with maturities 1 and 5 years.

Approaching the time of default, a spread widening and inversion of both senior and subordinated term structures can be observed. Calibrating the introduced Cox-Ingersoll-Ross model to AIB CDS quotes for each week independently for several maturities leads to the resulting implied recoveries and 5-year default probabilities shown in Fig. 2.

¹⁰ In the current version of the upcoming ISDA supplement, subordinated CDS may also settle without effecting senior CDS. However, so far either both or none settles.

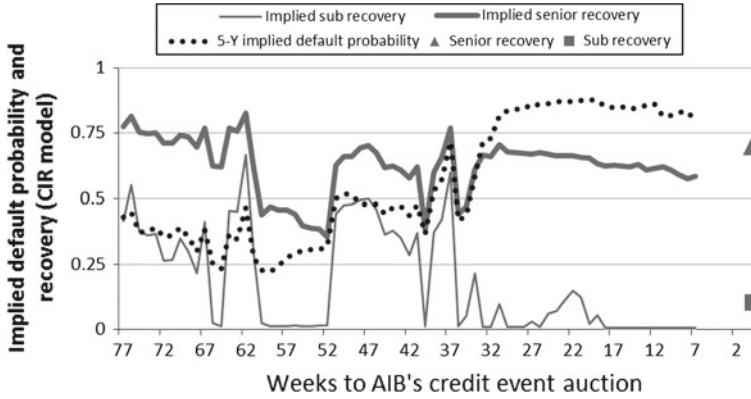


Fig. 2 Weekly calibration results for the CIR model applied to CDS spreads of AIB before its default in June 2011

Implied senior and subordinated recoveries and implied default probabilities vary substantially over time. One reason is that term structure shapes and general spread regimes also vary unusually strong from week to week, since AIB is in distress. Furthermore, there are co-movements of the 5-year implied default probability and the implied recoveries. This is caused by the fact that a (recovery) and θ (long-term default intensity) have a similar effect on long term CDS spreads. Assuming $\lambda_t \equiv \theta$ for all $t > t^* > 0$, the fair long term spread can be approximated via

$$s_T \approx c_0 + (1 - ae^{-\theta})\theta, \quad \text{for all } T > t^*, \quad (4)$$

where $c_0 \in \mathbb{R}$ is constant. Hence, using the above approximation for a given spread s_T , the optimal recovery parameter a^* can be seen as a function of the long term default intensity, denoted as $a^*(\theta)$. This entails the existence of a continuum of parameter values $(\kappa^*, \theta, \sigma^*, \lambda_0^*, a^*(\theta))$, $\theta > 0$, which all generate a comparable long term spread and thus a similar RMSE. Consequently, a minor variation in the quoted spreads might cause a substantial change in the resulting optimal parameters and thus in the implied recovery and implied probability of default. This is referred to as identification problem.

The following section contains a framework to circumvent this identification problem.

3.2 Pure Recovery Model

Two CDS contracts with the same reference entity and maturity, but differently ranked reference obligations, face the same default probabilities, but different recoveries. The general idea of the “pure recovery model” goes back to Unal et al. [15] and

Schläfer and Uhrig-Homburg [16]. The approach makes use of this fact by considering the fraction of two differently ranked CDS spreads, which is then free of default probabilities. Hence, spread ratios are considered and modeled and default probabilities can be neglected. A comparable approach is outlined in Doshi [17]. Let s^{sen} and s^{sub} denote the fair spreads of two CDS contracts referring to senior and subordinated debt. The basic idea can be illustrated using the credit triangle formula from Eq. (1), i.e.,

$$\frac{s^{\text{sen}}}{s^{\text{sub}}} \approx \frac{(1 - \Phi^{\text{sen}})\lambda}{(1 - \Phi^{\text{sub}})\lambda} = \frac{1 - \Phi^{\text{sen}}}{1 - \Phi^{\text{sub}}}. \quad (5)$$

Under simplified assumptions the ratio of two different types of CDS spreads is a function of the recoveries Φ^{sen} and Φ^{sub} . In case of the credit triangle formula, for instance, the underlying assumptions include independence of λ and Φ . The crucial point is to find a suitable and sophisticated model, such that this fraction again only contains recovery information. Implied recoveries are then extracted by calibrating fractions of senior and subordinated spreads. We propose a model that allows for time variation in Φ but no dependence on the default time τ .

In a first step, a company-wide recovery rate X_T is defined, i.e., a recovery for the whole company in case of a default until T , where T_{max} is the maximum of all instruments' maturities which should be captured by the model. Suppose $\mu_0 \in (0, 1)$, $\mu_1 \in (-1, 1)$, and $\mu_0 + \mu_1 \in (0, 1)$. Furthermore, let $v \in (0, 1)$. For a certain maturity $T_{\text{max}} > T > 0$, X_T is assumed to be Beta-distributed with the following expectation and variance:

$$\mathbb{E}_{\mathbb{Q}}[X_T] = \mu(T) := \mu_0 + \mu_1 \sqrt{T/T_{\text{max}}}, \quad (6)$$

$$\text{Var}_{\mathbb{Q}}[X_T] = \sigma^2(T) := v[\mu(T) - \mu(T)^2]. \quad (7)$$

The Beta distribution is a popular choice for modeling stochastic recovery rates, since it allows for an U-shaped density on $[0, 1]$ that is empirically confirmed for recovery rates. The above parameter restrictions assure that a Beta distribution with $\mathbb{E}_{\mathbb{Q}}[X_T]$ and $\text{Var}_{\mathbb{Q}}[X_T]$ as above actually exists. The square-root specification allows for a higher differentiation between maturity specific recoveries near $T = 0$, a phenomenon which is also widely reflected in CDS market term structures. Overall, this company-wide recovery distribution varies in time without depending on τ . In a second step, the seniority specific recoveries Φ_T^{sen} and Φ_T^{sub} are defined as functions of X_T . In legal terms, such a relation is established via a pecking order, defined by the Absolute Priority Rule (APR): In case of a default event, any class of debt with a lower priority than another will only be repaid if all higher ranked debt is repaid in full. Furthermore, all claimants of the same seniority will recover simultaneously, i.e., they receive the same proportion of their par value. Let d_{sec} , d_{sen} , and d_{sub} denote the proportions of secured, senior unsecured, and subordinated unsecured debt, respectively, on the balance sheet of a company at default, such that $d_{\text{sub}} + d_{\text{sen}} + d_{\text{sec}} = 1$. Figure 3 illustrates the APR.

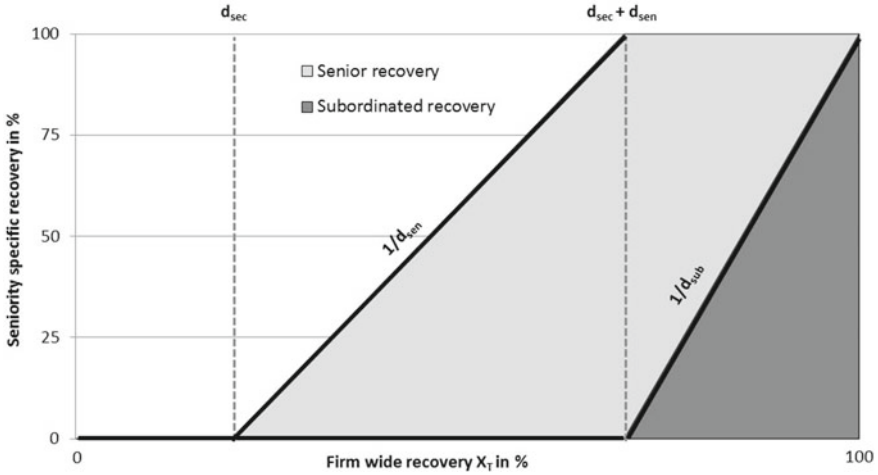


Fig. 3 Absolute priority rule: seniority specific recoveries depend on the stochastic firm-wide recovery and the debt structure of the company

The parameters d_{sub} , d_{sen} , and d_{sec} determine, which proportion of X_T is assigned to senior and subordinated debt holders if a default occurs. Motivated by the linkage of bonds and CDS in the auction mechanism, Φ_T^{sen} and Φ_T^{sub} are also assumed to be the appropriate CDS recoveries. Note, however, that in practice, APR violations often occur and are widely examined (see, e.g., Betker [18] and Eberhart and Weiss [19]). Using the APR rule, a general spread representation as in Eq. (2) as well as independence of Φ and τ , the recoveries are deterministic functions of the company-wide recovery X_T and the fraction of senior to subordinated CDS spreads is given by

$$\frac{s_T^{\text{sen}}}{s_T^{\text{sub}}} = \frac{1 - \int_{d_{\text{sec}}}^{d_{\text{sec}}+d_{\text{sen}}} \frac{x-d_{\text{sec}}}{d_{\text{sen}}} f_{p_T, q_T}(x) dx - \int_{d_{\text{sec}}+d_{\text{sen}}}^1 f_{p_T, q_T}(x) dx}{1 - \int_{d_{\text{sec}}+d_{\text{sen}}}^1 \frac{x-(d_{\text{sec}}+d_{\text{sen}})}{d_{\text{sub}}} f_{p_T, q_T}(x) dx}, \quad (8)$$

where $f_{p_T, q_T}(x)$ denotes the density of a $\text{Beta}(p_T, q_T)$ -distributed random variable. The variables p_T and q_T are linked to the parameters μ_0 , μ_1 , and v via Eqs. (6) and (7) and the first two moments of the Beta distribution. They are calibrated using the above formula, whereas the balance sheet parameters d_{sec} , d_{sen} , and d_{sub} are directly taken from quarterly reports. Instead of calibrating a single-spread curve, the calibration is carried out by matching theoretical fractions $s_T^{\text{sen}}/s_T^{\text{sub}}(\mu_0, \mu_1, v)$ in Eq. (8) for a set of several maturities to their market counterparts $s_T^{M, \text{sen}}/s_T^{M, \text{sub}}$, i.e.

$$(\mu_0^*, \mu_1^*, v^*) := \argmin \sqrt{\frac{1}{|\mathbb{I}|} \sum_{T \in \mathbb{I}} \left(s_T^{M, \text{sen}}/s_T^{M, \text{sub}} - s_T^{\text{sen}}/s_T^{\text{sub}}(\mu_0, \mu_1, v) \right)^2}.$$

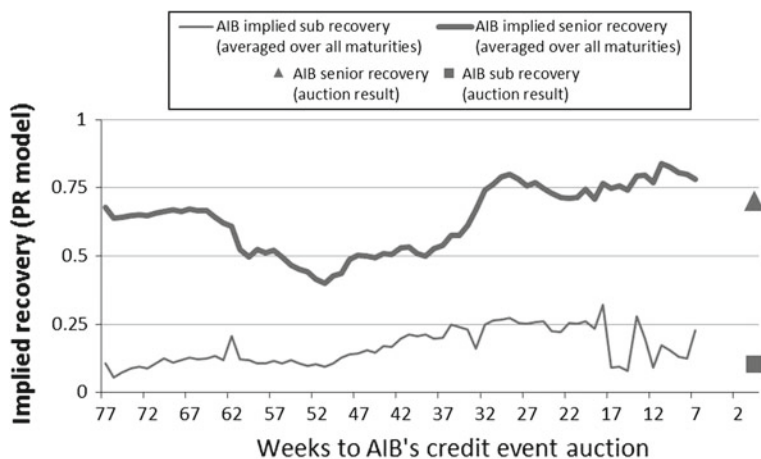


Fig. 4 Weekly calibration results for the pure recovery model applied to CDS spreads of AIB before its default in June 2011

The resulting risk-neutral implied distribution of the company-wide recovery can be translated into risk-neutral seniority specific recovery distributions by applying the APR rule. Furthermore, we could proceed to use this implied recovery result and extract implied default probabilities in a second step.

Calibrating the pure recovery model to senior and subordinated spreads from AIB (see Fig. 1) before its default yields implied recoveries for senior and sub debt, averaged over all maturities as displayed in Fig. 4.

As opposed to the Cox–Ingersoll–Ross model, the resulting recoveries do not exhibit sudden jumps, but are more stable over time. Only during the last weeks before default (weeks 17 to 7), particularly the subordinated recovery fluctuates. However, this is related to the significant movements of the market spreads and not originated by an identification problem among the parameters. Moreover, both senior and subordinated recoveries are in line with the later auction results, at least with respect to their proportional relation.

4 Conclusion and Outlook

Extracting implied recoveries and implied default probabilities in a risk-neutral setting tends to generate instable parameter estimates. The identification problem among long-term default probabilities and recovery rates is not limited to the presented CIR model, but can also be observed, e.g., in jump-to-default equity models such as the one proposed in Das and Hanouna [13]. We illustrated one way to circumvent the problem by reducing the calibrated expression to a form, where only recovery-related parameters appear. This is possible by considering instruments with different

seniorities, such as senior and subordinated CDS.¹¹ Furthermore, the extracted risk-neutral recoveries are more in line with the observed final auction results. Generally, further instruments, e.g., loans or the recently more popular contingent convertibles could be used in a similar way.

Acknowledgments The authors would like to thank Michael Hünseler, Rudi Zagst, and an anonymous referee for their valuable remarks on earlier versions of the manuscript.

Open Access This chapter is distributed under the terms of the Creative Commons Attribution Noncommercial License, which permits any noncommercial use, distribution, and reproduction in any medium, provided the original author(s) and source are credited.

References

1. Altman, E.I., Kishore, I., Vellore, M.: Almost everything you wanted to know about recoveries on defaulted bonds. *Financ. Anal. J.* **52**(6), 57–64 (1996)
2. Altman, E.I., Brady, B., Resti, A., Sironi, A.: The link between default and recovery rate: theory, empirical evidence and implications. *J. Bus.* **78**(6), 2203–2227 (2005)
3. Bruche, M., González-Aguado, C.: Recovery rates, default probabilities and the credit cycle. *J. Bank. Financ.* **34**(4), 754–764 (2010)
4. De Spiegeleer, J., Van Hulle, C., Schoutens, W.: *The Handbook of Hybrid Securities*. Wiley Finance Series. Wiley (2014)
5. Schönbucher, P.J.: *Credit Derivatives Pricing Models*. Wiley Finance Series. Wiley, Canada (2003)
6. Haworth, H.: A guide to credit event and auctions (2011). <http://www.credit-suisse.com/researchandanalytics>
7. Creditex and Markit. Credit event auction primer (2010). <http://www.creditfixings.com>
8. Creditex and Markit. Credit event fixings (2013). <http://www.creditfixings.com>
9. Jankowitsch, R., Pullirsch, R., Veža, T.: The delivery option in credit default swaps. *J. Bank. Financ.* **32**(7), 1269–1285 (2008)
10. Du, S., Zhu, H.: Are CDS auctions biased? Working Paper, Stanford University (2011)
11. Jaskowski, M., McAleer, M.: Estimating implied recovery rates from the term structure of CDS spreads. Tinbergen Institute Discussion Papers 13–005/III, Tinbergen Institute (2012)
12. Madan, D.B., Bakshi, G.S., Zhang F.X.: Understanding the role of recovery in default risk models: empirical comparisons and implied recovery rates. FDIC CFR Working Paper No. 06; EFA 2004 Maastricht Meetings Paper No. 3584; FEDS Working Paper; AFA 20004 Meetings, 2006
13. Das, S., Hanouna, P.: Implied recovery. *J. Econ. Dyn. Control* **33**(11), 1837–1857 (2009)
14. Höcht, S., Zagst, R.: Pricing credit derivatives under stochastic recovery in a hybrid model. *Appl. Stoch. Models Bus. Ind.* **26**(3), 254–276 (2010)
15. Unal, H., Madan, D., Güntay, L.: Pricing the risk of recovery in default with absolute priority rule violation. *J. Bank. Financ.* **27**(6), 1001–1218 (2003)
16. Schäfer T.S., Uhrig-Homburg, M.: Estimating market-implied recovery rates from credit default swap premia. University of Karlsruhe Working Paper, 2009
17. Doshi, H.: The term structure of recovery rates. McGill University Working Paper (2011)

¹¹ Note that there is an ongoing discussion regarding the admission of credit events, which are only binding for subordinated CDS. Such a possibility is explicitly excluded in the proposed pure recovery model.

18. Betker, B.L.: Management's incentives, equity's bargaining power and deviations from absolute priority in Chapter 11 bankruptcies. *J. Bus.* **68**(2), 161–183 (1995)
19. Eberhart, A.C., Weiss, L.A.: The importance of deviations from the absolute priority rule in Chapter 11 bankruptcy proceedings. *Financ. Manag.* **27**(4), 106–110 (1998)